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A BIBLIOGRAPHY OF FREE BOUNDARY PROBLEMS.(U)

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MRC Technical Summary Report # 1793

A BIBLIOGRAPHY OF FREE BOUNDARY PROBLEMS

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MATHEMATICS RESEARCH CENTER

A BIBLIOGRAPHY OF FREE BOUNDARY PROBLEMS

Colin W. Cryer

Technical Summary Report #1793
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ABSTRACT

A free boundary problem is a (steady-state) boundary value problem involving differential equations on domains parts of whose boundaries, the free boundaries, are unknown and must be determined as part of the solution. Free boundary problems occur in all branches of continuum mechanics. A typical free boundary problem is the problem of a water jet in air, where the unknown free boundary is the water/air interface.

The bibliography contains about 3300 references, most of which are classified according to one or more of approximately 1200 subjects. This report contains a description of the bibliography and a list of the subjects, which are divided into three main groups: I, Types of free boundary problems; II, Mathematical methods; and III, Approximation methods. Listings by author and subject are given in appendices; these appendices are on microfiche, but paper copies may be obtained by writing to the Mathematics Research Center.

AMS(MOS) Subject Classification - 31-02, 35J99, 35R99, 45-02, 49-02, 65-02, 65P05, 65R05, 73-02, 76-02, 78-02, 80-02, 85-02, 86-02.

Key Words - Free boundary problems, Fluid mechanics, Porous flow, Mechanics of solids, Heat conduction and diffusion, Electromagnetism, Gravitation, Mathematical methods, Numerical methods.

Work Unit Number 3 - Applications of Mathematics

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A BIBLIOGRAPHY OF FREE BOUNDARY PROBLEMS

Colin W. Cryer

1. Description of the bibliography. A free boundary problem (FBP; plural, FBPS) is a (steady-state) boundary value problem involving differential equations on domains parts of whose boundaries, the free boundaries (FB; plural FBS), are unknown and must be determined as part of the solution.

FBPS arise in a great variety of contexts, and the literature on the subject is scattered throughout many different disciplines. The present bibliography has been compiled in an attempt to bring together all the important results on FBPS. It is our hope that this survey will reduce the duplication of effort and stimulate the cross-fertilization of ideas among the different disciplines concerned with FBPS.

The bibliography contains about 3300 references most of which are classified according to one or more of approximately 1200 subjects.

In the remainder of this section we make some general remarks about the bibliography. In section 2 we describe the approach used to classify the different types of FBPS in continuum mechanics. In section 3 we give abbreviations and conventions. Finally, section 4 contains a list of the subjects. Listings by author and by subject are given in appendices; these appendices are on microfiche, but paper copies may be obtained by writing to the Mathematics Research Center.

References for the bibliography were obtained in the following ways:

(i) By searching the recent issues of about forty journals which contain a relatively large number of papers on FBPS. The following journals contained an especially large proportion of papers on FBPS:

Archive for Rational Mechanics and Analysis

Communications on Pure and Applied Mathematics

Journal of Fluid Mechanics

Physics of Fluids

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Proceedings American Society of Civil Engineers (Division of Engineering Mechanics, Division of Hydraulics and Division of Soil Mechanics)

Water Resources Research

(ii) By following up the bibliographies of books and papers.

(iii) By writing to a large number of authors requesting reprints of their recent work.

(iv) By using the following abstracting services:

Applied Mechanics Reviews

Computing Reviews

Contents of Contemporary Mathematical Journals

Dissertation Abstracts International

Mathematical Reviews

Science Citation Index

(v) By browsing in libraries and bookstores, and chatting to colleagues.

Almost every reference was consulted and copies were obtained of almost all papers and many of the books.

The literature on FBPS is enormous. To quote a typical example, between 1965 and 1968 over five hundred papers were published on the motion of bubbles and drops. In consequence, we had to be selective in deciding which references to include. The following criteria were used in determining which references should be included:

(i) Papers describing heuristic approximation methods, papers describing experimental results, and papers using the well-known hodograph method are not usually included.

(ii) All papers known to us giving existence or uniqueness theorems are included.

(iii) All papers known to us describing numerical methods are included.

(iv) Whenever possible, a worker in the field is represented by at least one paper.

(v) When an author has himself presented a summary of his results, perhaps in the form of a book, sometimes only the summary and particularly note-[illegible]hy papers are included.

(vi) When a series of papers forms a natural sequence - perhaps giving first-order, second-order, and higher-order approximations - often only the last paper in the sequence is included.

(vii) Each approach and each type of problem known to us is represented by at least one reference.

(viii) Papers containing interesting new ideas are included even when not directly concerned with FBPS when it is felt that the results have possible future applications to FBPS.

(ix) Preference is usually given to recent work, but papers of historical interest are included.

The over-riding criterium in selecting references was that the bibliography should remain comprehensible and manageable to us and useful to the reader.

In constructing the list of subjects, our goal was to devise a systematic classification which was so organized that the information could be readily retrieved. The classification which we use is rather formidable, but it is hoped that it will make it possible for the reader interested in a particular problem to locate precisely the information of interest to him. There are three broad classes of subjects: I, Types of free boundary problems; II, Mathematical methods; and III, Approximation methods. The classification of FBPS in continuum mechanics presented especial difficulties which are discussed in section 2. Most references of the bibliography are followed by a list of cross-references to the subjects with which they are concerned.

Of the major scientific languages other than English, we read German fluently and read French, Italian, and Russian with difficulty. This has affected the choice of references to some extent, and has obviously affected our ability to comprehend certain references, particularly long references in Russian; such references are therefore cross-referenced less frequently than they deserve.

The bibliography has been constructed in conjunction with a series of surveys on FBPS which are appearing as Mathematics Research Center Technical Summary Reports. The following comments arise:

(i) Two surveys have already appeared, namely Technical Summary Report #1657 on porous flow FBPS which corresponds to I.4 in the subject listing, and Technical Summary Report #1693 on trial free-boundary methods, which corresponds to III.3 in the subject listing. References quoted in these two surveys are listed in the present bibliography, and some new references have also been added.

(ii) Many sections of the other surveys have already been written. Some of the terminology used in the list of subjects and some of the cross-references made will perhaps seem obscure to the reader and may only become clear when the corresponding surveys appear.

The bibliography was begun during a sabbatical spent in 1972 at Oxford University with the support of the United Kingdom Science Research Council and the National Science Foundation, and would not have been written had it not been for the interest in FBPS of Professor L. Fox of the Oxford University Computing Laboratory.

The National Science Foundation (through Contract No.: GJ-33096) provided substantial help which enabled us to work full-time on the project for nine months and to meet the costs of xeroxing and typing. We are particularly grateful in that additional support was given when the original estimates of time and cost proved to be hopelessly wrong.

Thanks are due to the staff of the Radcliffe Science Library (Oxford), and the Memorial, Physics, and Engineering Libraries (Madison), for their help and forbearance.

We would welcome information about errors and omissions, as well as reprints of work on FBPS. These would be incorporated in a revised edition of this bibliography which we hope to prepare.

2. Classification of FBPS in continuum mechanics. There is no completely satisfactory method of classifying FBPS in continuum mechanics. The approach followed is:

(i) To classify first according to the broad subdivisions of continuum mechanics such as fluid mechanics.

The classification is based upon the "nontrivial" field equations occuring. We illustrate this by means of some examples. The problem of water waves subject to gravity involves both fluid mechanics and gravitation, but the gravitational field is given so that the problem is classified under fluid mechanics. The problem of a rotating self-gravitating fluid mass also involves both fluid mechanics and gravitation, but in this case the fluid velocity is usually prescribed, so that the problem is classified under gravitation. Problems involving a plasma in equilibrium under a magnetic field are often described as magnetohydrodynamic problems but in many cases the hydrodynamic equations are trivially satisfied and in such cases the problems are classified under electromagnetism.

Problems involving two nontrivial fields are treated in one of two ways:

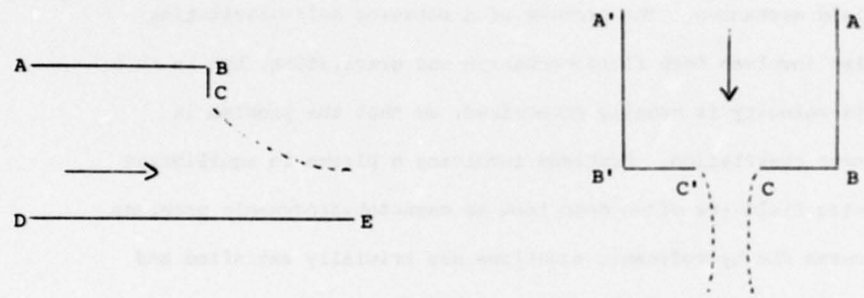
(a) If the problem can be regarded as a generalization of a FBP in one area of continuum mechanics (the primary area), then the problem is classified under the primary area. For example, porous flow FBPS can be generalized by considering electrical effects: the resulting electrokinetic porous flow problems are classified under porous flow problems.

(b) If a problem inherently involves two or more areas of continuum mechanics, then it is classified under coupled field problems. For example, the flow of a blood corpuscle through a capillary is quite different from problems in either fluid dynamics or elasticity and is classified as an elasto-hydrodynamic problem in the chapter on coupled field problems.

When a problem is a combination of two FBPS, it is classified according to the primary problem. For example, the problem of a cavitating hydrofoil under a water surface is classified under cavities because this is of primary interest.

(ii) To classify next according to the geometry of the problem.

A major difficulty is that by interchanging axes or invoking symmetry a problem can be transformed into an apparently different problem. Consider, for example, the horizonatal flow of a fluid out of an obstructed channel (Figure 1a).



(a) Horizontal flow from a channel

(b) Vertical jet flow

Figure 1: Equivalent problems

When gravity is neglected the boundary DE is an axis of symmetry and the problem is equivalent to a vertical jet (Figure 1b). However, when gravity is taken into account or when the fixed boundaries are slightly changed, the two problems are no longer equivalent.

Axisymmetric problems are classified according to their plane equivalents. For example, the plane equivalent of an axisymmetric is a plane jet, but the plane equivalent of an axisymmetric stream flowing around a wire is a stream flowing down a plane wall. This approach is quite appropriate because of the close relationship between corresponding plane and axially symmetric problems.

(iii) <u>To classify finally according to the governing equations and boundary conditions</u>.

For example, the vertical jet shown in Figure 1b is considered for various governing equations (inviscid flow, viscous flow, etc.) and various boundary conditions (no gravity, gravity, surface tension, etc.)

The method of classification described above (like any other) cannot eliminate all anomalies. The same mathematical problem may arise in two different areas of continuum mechanics. Should the problem of a hydrofoil cavity underneath a water surface be classified under cavities or under water waves? Does a problem have a symmetrical equivalent? The reader must be prepared to look for a problem under more than one heading.

3. Abbreviations and conventions.

The bibliography is arranged alphabetically. A prefix is treated as part of the name so that von Mises is listed under V and not under M. Names beginning with Mac and Mc are listed under MAC and MC, respectively.

The transliteration of Cyrillic names is handled as follows. If a reference has been translated, then the spelling of the translation is used. If a reference has not been translated then a review is quoted and the spelling of the review is used. In some cases this approach leads to an author's name appearing with different spellings. When this would appear to create misunderstanding, alternative spellings are given in brackets. (For many years we were unaware that the famous Joukowsky in fluid mechanics was the same person as the famous Zhukovsky in porous flow.)

When a reference has appeared in two essentially equivalent forms this is indicated by means of an "=" ; the date used is usually the earliest date.

Proceedings and Symposia are listed under the editors when known. Otherwise, they are listed, not very systematically, under "Proceedings", "Symposium", or the conference title.

When a reference has been reported to contain errors, the source of this information is indicated. Occasionally, general observations about a reference are also included.

In general, each reference is followed by a list of cross-references to the subjects to which it is relevant. Some very important references are referred to so often that the list of cross-references is very long. In such cases the list of references is given in the author listing but is omitted in the subject listings.

As mentioned in section 1, the subjects listed under I.4 (porous flow problems) and III.3 (trial free-boundary methods) correspond to two published surveys but also include some new references. These "new" references are identified by an "N" following the cross-reference, and in the subject listing they are listed separately following the "old" references.

The following abbreviations are used:

U	:	Unknown or Unclassified
TA, TAA, etc.	:	To appear, to appear (a), etc.
CR	:	Computing Reviews
DA	:	Dissertation Abstracts International
MR	:	Mathematical Reviews
N	:	New references
**	:	List of cross-references omitted in subject listing.

Theses which are listed in Dissertation Abstracts are followed by their order number.

4. Author Index and Subject Index

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A free boundary problem is a (steady-state) boundary value problem involving differential equations on domains parts of whose boundaries, the free boundaries, are unknown and must be determined as part of the solution. Free boundary problems arise in all branches of continuum mechanics. The bibliography contains about 3300 references most of which are classified according to one or more of approximately 1200 subjects. Listings by author and by subject are given in appendices; these are on microfiche, but paper copies can be obtained by writing to the Mathematics Research Center.

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